OBTAINING SIMPLIFIED FINITE ELEMENT MODELS BY MINIMIZATION OF RESIDUALS OF STATIC AND DYNAMIC RESPONSES

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Abstract. The study deals with the problem aming to formaly reduce a complex finite element structural model to a simpler one. As a sample task, the reduction of a girder model to the simpler equivalent membrane model has been investigated. The coincidence of model displacements at given loading condirtions is employed as a criterion of mutual adequacy of the two models. Both static and dynamic displacements at selected reference points have been used in the expression of the penalty-type target function, the minimum of which indicates the best fit between the original and reduced models. The target function has been miminized by using the geometrical and physical parameters of a typical membrane element as optimization variables. The calculations have been performed in MATLAB environment. The validity of obtained parameters of the membrane model has been tested by investigate the original and reduced structures of different geometrical shapes at complex loadings.

Keywords: Finite element models, reduction, parameter identification

1 INTRODUCTION

Finite element techniques in principle enable to analyse structures of any level of complexity, including their essentially non-linear behaviour, peculiarities of internal texture, etc. [4]. However, highly adequate models are often generated on expense of very complex structures, huge dimensionalities and internal interactions, which require very large and often prohibitive amounts of computational resources. Building simplified (reduced) computational models is a common practice enabling to obtain solutions with practically acceptable costs.

As an example, a woven textile structure can be represented by using models of different levels of detalization. A woven structure composed of shell elements[1], simpler and more efficient combined particles model[2], orthotropic membrane models[3], have been employed in order to represent the dynamic behavior of textile cloths under conditions of mechanical impact and penetration. A special attention and prospectives deserve models, in which central and distant zones of the same structure are presented by different models. As in [3], the zone of ballistic interaction of the textile structure has been modeled by using the complex contact model of a woven structure, meanwhile the distant zones have been presented by membrane elements. The coupling between the zones has been implemented by means of the tie constraint. The main purpose of this combination was to implement the "almost infinite" surrounding.

As a rule, such combined models are obtained by using a lot of engineering intuition and basing on profound knowledge of physical properties of the investigated phenomena. More regular approaches are necessary, which enable to synthesize simplified or reduced models of internally complex structures. The parameters of the reduced model can be adjusted by performing the minimization of error functions, quantitatively indicating the non-coincidence of the response between the simplified and reference models. An alternative approach can be based on neural network techniques in order to synthesize the models exhibiting the required structural response [6].

In this work, we demonstrate a procedure and results of synthesis of the continuous membrane model, which imitates the behavior of the girder structure under static, as well as, dynamic loadings.

2 PROBLEM FORMULATION

The analyzed source structure is a 2D girder composed of tiny beam elements, and the approximating reduced model is a planar membrane. The girder consists of rods of uniform width and thickness. It is necessary to find the parameters of the equivalent orthotropic membrane. Assume that the membrane model presents a satisfactory approximation of the girder if the displacements of the nodes at the same loading are obtained nearly the same by using both models.

Consider rectangular plate and rectangular girder having identical dimensions. The geometry of the girder is described by width *h*, thickness *b* and spacing *N*. Physcial parameters used in the small displacement elasticity model are Young's modulus E and mass density ρ. The membrane model is characterized by thickness s_m and orthotropic material parameters: Young's modules E_{m11}, E_{m22}, Poisson's ratios ν_{m12}, ν_{m21}, shear module G_{m12} and mass density ρ_m.

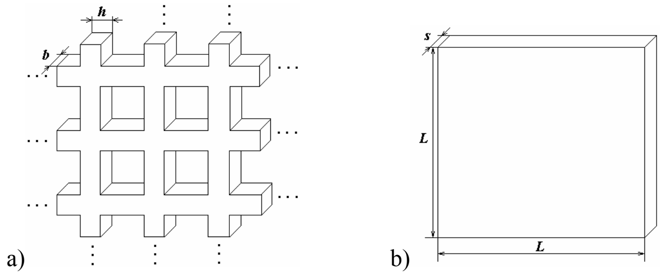


Figure 1. The girder (a) and equivalent steel membrane (b).

Membrane parameters ν_m, E_m, G_m and s_m have to be established, which enable the membrane to exhibit the same or similar behaviour in terms of displacements of respective element nodes at a given loading.

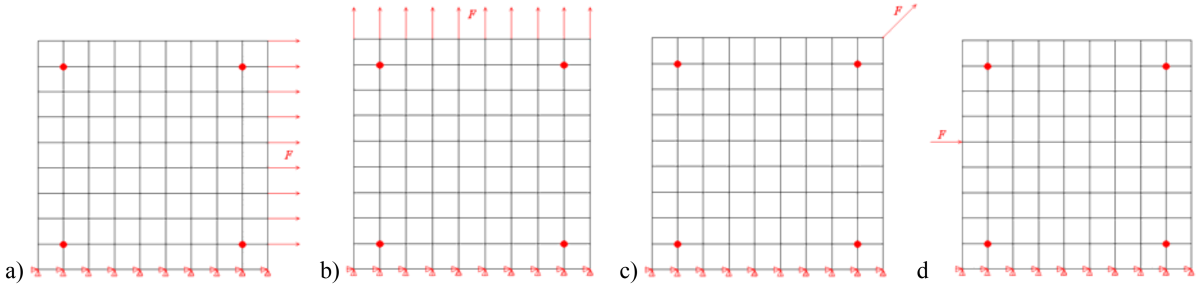


Figure 2. The finite element models: a) 1st analysis model; b) 2nd analysis model; c) 1st test model; d) 2nd test model.

As a measure of quality of the approximation of the girder model by equivalent membrane model we employ the minimum of a penalty-type target function expressed as a sum of squares of differences between the displacements of corresponding nodes of each model. The static as well as dynamic behaviour of the two structures has been analyzed. The schemes of two static loading cases are presented in Fig.2 a) and b), where both structures have been exposed to static load *F* and the differences of displacements of 4 selected nodes have been included into the penalty function expression.

The obtained parameters of the equivalent membrane shall be tested by using several freely selected test models(loading cases) two of which are presented in Fig.2 c) and d).

In the case of dynamic analysis, the differences between displacements of nodes are minimized at selected time moments. The analysis has been performed by using ANSYS and MATLAB software. The displacements obtained in ANSYS have been used when forming the target function, which subsequently has been minimized by employing MATLAB function FMINCON().

3 ANALYSIS OF RESULTS

3.1 Static analysis

Further, the analysis of the statics of selected girder as well as parameters of membrane resulted in the optimization, are presented. In order to facilitate the optimization problem assume $E_g = E_{m11} = E_{m22}$, $\nu_{m12} = \nu_{m21} = 0$, $\rho_g = \rho_m$ and $h = b$. Assume the girder rods being thin enough to maintain the mechanical features of the girder:

$$\frac{1}{20} \le \frac{b}{L_m} \le \frac{1}{8},\ L_m = \frac{L}{N} \ \Rightarrow\ \frac{L}{20 \cdot N} \le b \le \frac{L}{8 \cdot N}, \qquad (1)$$

where L is the side length of the rectangular element and N – number of divisions of the side. In this example, numbers of divisions of the plate and the girder are selected the same, however, generally the grid spacing may be much smaller than the side length of the membrane element.

Consider the models in Fig.2. In the first load case (LC1) (Fig.2a), all nodes of the bottom side are fixed meanwhile all nodes of the right hand side are exposed to forces imitating distributed loading along *Ox* direction. The second load case(LC2) model (Fig.2b) the top side is exposed to distributed loading along *Oy* direction.

The target function reads as follows:

$$T(\vec{P},\vec{Q}) = \frac{\sum_{i=1}^{n}(\vec{p}_1^{\,i} - \vec{q}^{\,i})^2}{\sum_{i=1}^{n}(\vec{p}_1^{\,i})^2 + \sum_{i=1}^{n}(\vec{q}^{\,i})^2} + \frac{\sum_{i=1}^{n}(\vec{p}_2^{\,i} - \vec{q}^{\,i})^2}{\sum_{i=1}^{n}(\vec{p}_2^{\,i})^2 + \sum_{i=1}^{n}(\vec{q}^{\,i})^2}, \qquad (2)$$

where p_1^i, p_2^i are the vectors of i-node displacements of the 1st and 2nd models, $n=(N+1)^2$ - total number of the nodes of each model.

After the minimization of (2) we obtained the relationship of optimum thickness s_m of the equivalent membrane against the girder rod thickness b and against the shear module G_m. By applying the least squares approximation (LSA), the square and linear relationships between the parameters optimization variables has been established as in Fig. 3.

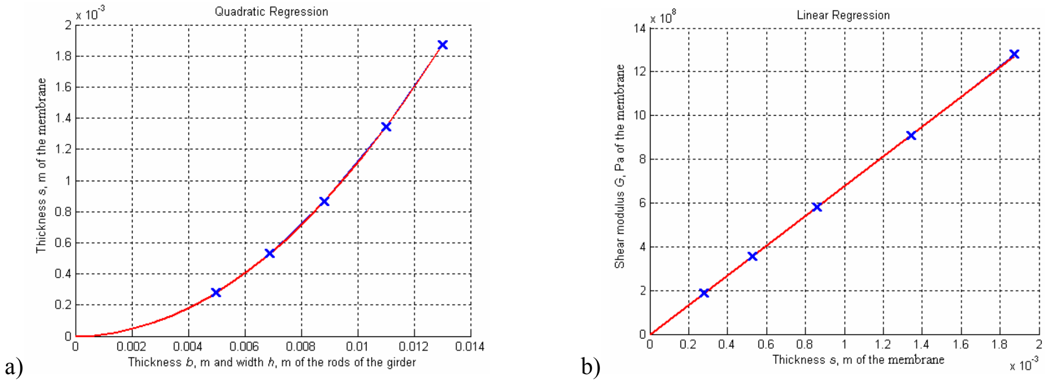


Figure 3. The pairs of optimal parameters (blue points) of the girder the equivalent membrane and the regression curves (red lines), (a) – square fit; (b) – linear fit.

The analytical expressions of the regression curves presented in Fig.3 read as

$$s_m = s(N,b) = b^2\cdot(1{,}1275 + 1{,}1088\cdot N),$$
$$G_m = G(N,b) = N\cdot 10^3(293{,}09 - 3{,}6260\cdot N) + b\cdot 10^8(1{,}6818 - 2{,}0013\cdot N) + N^2\cdot b^2\cdot 10^{10}(9{,}4502) \quad (3)$$

The estimation of derived formulas (3) against calculated pairs of optimum parameters at different values of grid parameter b and side division N is presented in Fig. 4.

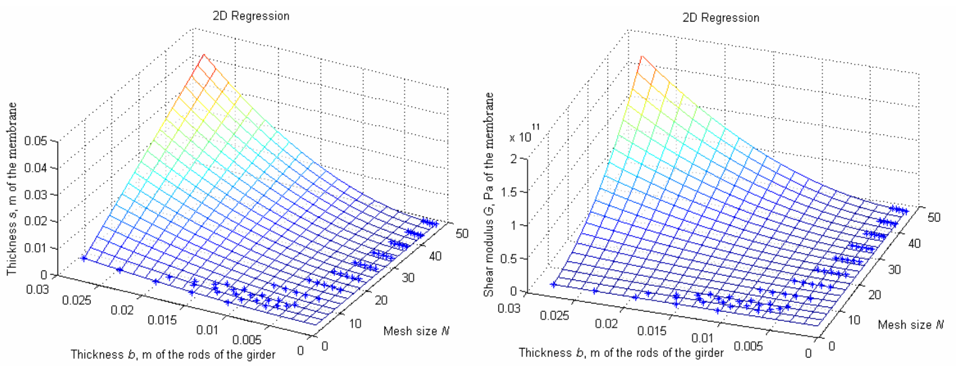


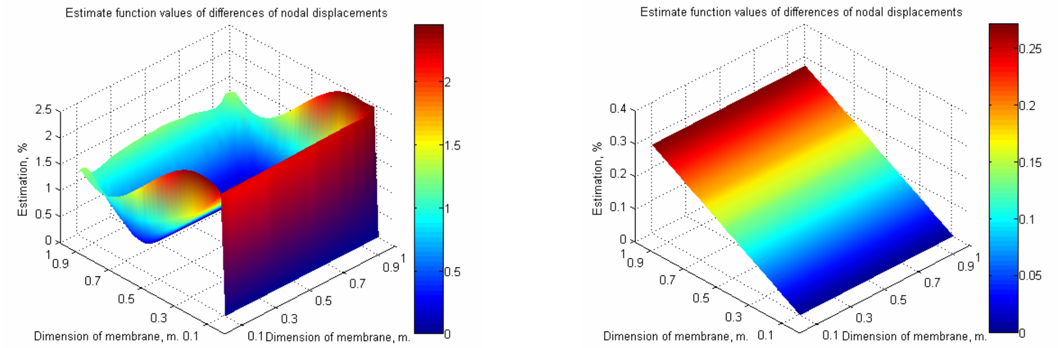
Figure 4. 2D regressions of depending parameters.

It follows from Fig.4 that the increase of the mesh division parameter N, makes each rod of the girder thinner, see formula (1). The corresponding values of the thickness of the equivalent membrane and its shear module tend to decrease. The deviations of calculated optimum values of the membrane parameters with respect to the obtained regression (3) have been evaluated as

$$\Delta_i = \frac{n\cdot\sqrt{(p_x^i - q_x^i)^2 + (p_y^i - q_y^i)^2}}{\sum_{i=1}^{n}(|q_x^i| + |q_y^i|)}, \quad (4)$$

where n – total number of nodes of each model, p_x^i, p_y^i are x and y displacements of node i of the membrane, q_x^i, q_y^i are x and y displacements of node i of the girder.

Further, the evaluation of the derived (3) dependencies is presented. The size of tested models was $N=38$, and the values b of the girder were chosen in accordance with the formula $b=\frac{L}{8\cdot N}$. The parameters of equivalent membrane have been calculated according to formula (3). The same loading of the model has been used in all investigated cases as in Fig.2 a) and b). The estimation of the deviations of membrane displacements from the reference displacements of the girder is presented in Fig. 5.

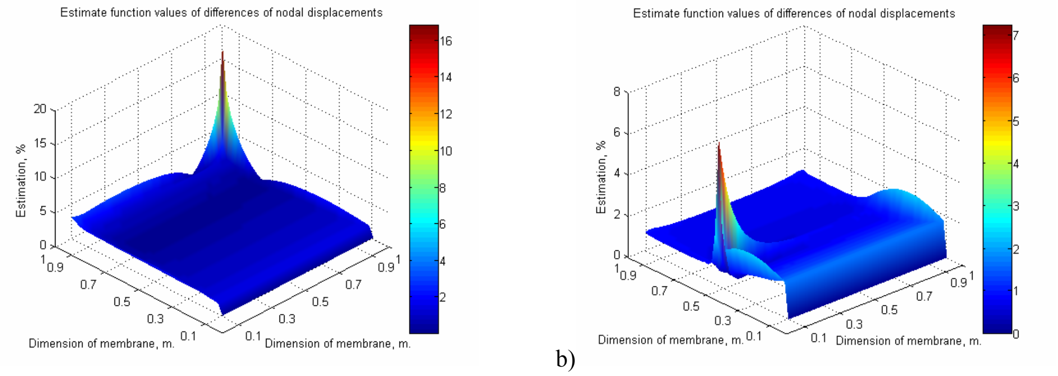


a) b)

Figure 5. The estimated values of differences of displacements of respective nodes of membrane and girder by using 1st (a) and 2nd (b) models(Fig.2).

The largest deviations of the first model (Fig. 5a) are at the nodes in the vicinity of the constrained side of the membrane. On the contrary, in the second model (Fig. 5b) the deviations at the nodes nodes in the vicinity of the constrained side of the membrane are the smallest.

The next numerical experiment is performed by loading the same girder and equivalent membrane by means of the force applied in the plane *xOy* at the corner at angle 45^0, (model Fig.2c) Fig. 6a). Figure 6b) is obtained using model (model Fig.2d) by applying the force at the mid-side node.

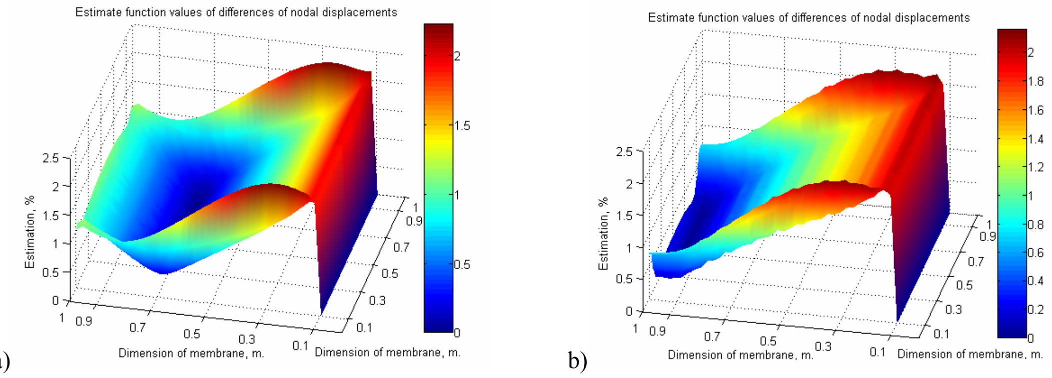


a) b)

Figure 6. The estimated values of differences of displacements of respective nodes of membrane and girder with free selected models.

We have found the equivalent membrane for the selected girder by considering the extra loading cases (models Fig. 2c) and d)) and determined that the deviations of relevant nodes have increased up to 6 times. The maximum value of relative displacenment deviation between the two models was equal to approximately 16%. The biggest deviations of displacements appear at the nodes affected by the force. In order to reduce the deviations we should include the displacements of latter two models into the target function to be optimized.

The estimations of displacement differences between the two models at first loading case is presented in Fig. 7a, b for the coincident (*N=38*) and non-coincident (*N=24*) mesh.



a) b)

Figure 7. The estimated values of differences of displacements of respective nodes of eqivavent membrane and refernce girder at coincident meshing *N*=38 (a) and non-coincident meshing *N*=24 (b).

It can be observed that the maximum estimation values did not change, however, the estimations at individual nodes may change significantly (up to 6 times in this case).

3.2 Dynamic analysis

Here we extend the regression formulas determined in section 3.1 for the static analysis to the dynamic analysis. Time is introduced as continuous variable $t \in [0;T]$ and optimization is performed by using a new target function, obtained by integrating expression (2) over time. Now P and Q are three-dimensional matrices, in which nodal displacements are stored at all time steps t_k . The integration over time is performed numerically by using the 5th order Newton – Cottes quadrature formula [5].

The time law of the loading is read as $T \approx L\left(\frac{E_g}{\rho_g}\right)^{-\frac{1}{2}}$. We chose the time interval of the transient dynamic analysis equal to the time necessary for the longitudinal elastic wave to travel distance L equal to the side length of the model. The sine-pulse shaped time law of force F we assumed to have period $T_F \approx \frac{T}{2}$. Forces at individual nodes have been applied $F = \frac{2 \cdot F_{\max} \cdot \sin(\pi \cdot t)}{2 \cdot N - 1}$.

We select 8 reference nodes (Fig. 8), at which displacement time laws of both structures are compared against each other.

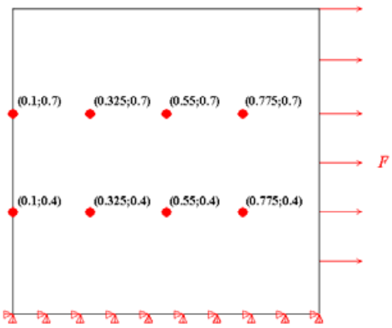


Figure 8. The reference nodes of the model.

In order to perform the minimization of the penalty type target function, the parameters of the membrane are calculated by using formula (3) derived for the static analysis case. In the simplest case we are using only one optimization variable as equivalent mass density ρ_m of the membrane. The mesh 48×48 in both structures is employed.

We use FMINCON() function in order to determine ρ_m^* value. The minimization process is shown in Fig. 9.

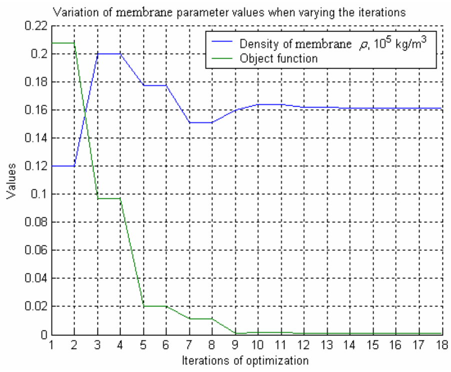
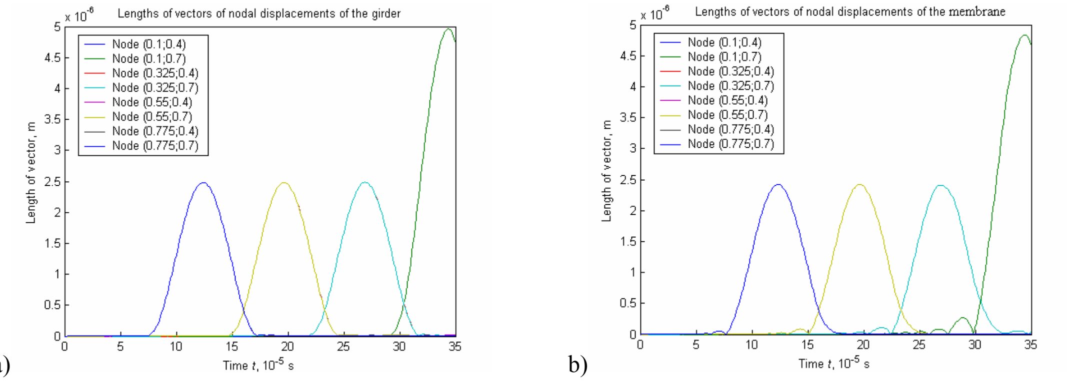


Figure 9. The dependence of membrane's optimizable parameter variation on iterations.

We get the equivalent membrane having the density $\rho_m^* = 16125 \frac{kg}{m^3}$, $\frac{\rho_m^*}{\rho_g} \approx 2{,}067$ ·

Fig.10 presents the magnitudes of displacements caused by the transient vibration processs (the wave traveling along *Ox* direction, Fig. 2a)) at several selected time moments in the girder(a) and equivalent membrane structure (b). The two graphs are practically equivalent to each other. Fig.11 presents errors of displacements of reference nodes.



a) b)

Figure 10. The magnitudes of displacements at several selected time moments in the girder (a) and equivalent membrane structure (b).

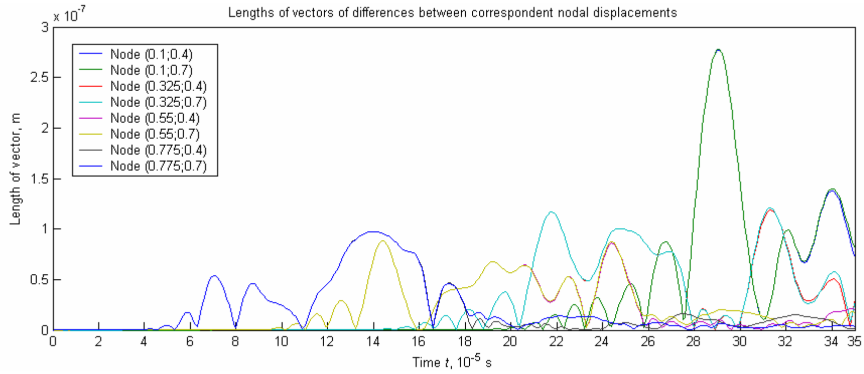


Figure 11. Differences between corresponding displacements of reference nodes of the girder and membrane model.

The obtained errors are quite small indicating that the regression formulas (3) are suitable for employing them for the dynamic analysis with only the equivalent density value of the membrane being adjusted properly.

4 CONCLUSIONS

A formal approach to the reduction of a complex finite element structural model to the simpler one has been proposed. The procedure is based on penalty type target function minimization in the space of parameters of the reduced model. As a sample task, the synthesis of the reduced equivalent continuous membrane model, which imitates the behavior of the girder structure, has been solved.

For the static analysis case the equivalent membrane parameter set has been determined at which the models worked satisfactorily in the case of coincident, as well as, non-coincident meshes of the reference and the reduced equivalent structure and at different loading configurations. Regression formulas for obtaining the equivalent parameters have been derived.

The equivalent parameters obtained for static analysis have been demonstrated to work also in the dynamic analysis case, provided that proper equivalent mass density value of the equivalent reduced structures is adjusted.

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